The Three-Door Problem as Narrative: *A Groundhog Day Perspective*

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In this article I shall explain how the solution to the [Monty Hall] *Three Door Problem* (the "Problem") is contingent upon its precise narrative presentation and can therefore be resolved by way of careful semantic and linguistic scrutiny.[1] Once the Problem is properly viewed from a narrative perspective, rather than a purely mathematical or statistical one, the apparent complexities vanish. Consider the following two versions of the Problem [hypothetical scenarios or "Hypos"]:

Hypothetical A *[The most common and, it is believed, the original]*:

You **are** a contestant on a game show. You are told you have an "option" to choose one of three doors [and, you are told, in one phrasing or another, ***you have proceeded to exercise that "option"***]. Behind one of the doors is a new car. Behind each of the other two doors are goats. The Host [Monty, from *Let's Make a Deal*] has then proceeded to open one of the doors to [***always***] reveal a goat. [End of factual narrative]

You are then asked by the Host whether you wish to change your initial door choice and switch to the other door [not opened by Host]. In other words, *should* you change - i.e. is there is a statistical advantage to switching from your initial door choice? Said yet another way, does a decision to switch increase (or decrease) your odds of winning, or neither [i.e. no statistical difference]?

Hypothetical B:

You ***will in the future be/find yourself as*** a contestant on a game show. You are told you **will have** an "option" to choose one of three doors [and, you are told, in one phrasing or another, ***you have proceeded to exercise that "option"***]. Behind one of the doors is a new car. Behind each of the other two doors are goats. The Host ***will then*** proceed to open one of the doors to [***always***] reveal a goat. [End of factual narrative]

You ***will then*** be asked by the Host… [and the remainder of the Problem and question is the same].

Each of the preceding scenarios contain subtle but crucial narrative distinctions, and it is the careful interpretation of these nuances and potential ambiguities upon which the correct answer to the Problem(s) turns.[2] For, as I will illustrate below, in at least one of foregoing narratives the so-called "Three-Door Problem" is really a ***two*-door** problem *disguised* as a Three-Door Problem.

More specifically, in the first narrative [A] - the one most in conformity with the original - the world of possible "prize-doors" sets is *always* and *necessarily* comprised of only two **from the outset**, and this is precisely the crux of the Problem.

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1 This article assumes the reader is generally familiar with the Problem. Otherwise, the following examples provide the necessary framework and additional information can be readily found online.

2 The following assumptions are also always implicit and true in all versions: the car is the desired prize; the Contestant does not know where the car is or is not; the Host always knows where the car is and is not; the car is always behind only one of the doors and is never moved; everyone speaks the truth; there are no intentional "tricks" or deceptions employed - i.e. this is a "straightforward" statistical problem.

The most frequent source of debate arises from erroneous assumptions drawn by listener Contestants who (understandably) fail to discern the admittedly ambiguous and subtle facts of the narrative. But, if one *carefully* scrutinizes the precise language of Hypo A it is - our should be - fairly easy to see that the actions of the Host are **empirically, factually and statistically** tantamount to the immediate removal or negation of [the world of] the one 'wrong-door' "choice" possibility – always and forever. And not just a post-game show round removal, but a persistent, *a posteriori* exclusion.

This is because in the precise wording of [and or implications which *must* be logically drawn from it] Hypo A, we are provided, as a necessary and indispensable fact, that one door "choice" - namely that of the Contestant [or the "you" in the Hypos] - ***has already been made*** *and* that the Host ***has already*** opened a "goat door". Thus, regardless of when or to whom Hypo A is presented, the **same exact** scenario, with the **same exact** facts [save for one, which is addressed below] will evidence itself time after time; hence what I refer to in this context as the "Groundhog Day effect". The **only** possible contingency - that is, the **only** factual variable - pertains to which of the ***other*** two [non-Host revealed] doors conceals - and must/will always conceal - the car.

Why is this the only possible contingency? Because we are told that the Contestant *has* - by the end of the factual narrative - made a choice of one particular door [so for the one being questioned or quizzed, a choice no longer exists]. Additionally, we are told that the Host *has* - by the end of the factual narrative - revealed a goat [thus, the "Host-door" will reveal and has *always* revealed a goat]. This can be analogically translated thusly, "Contestant **always** chooses door #1 and Host **always** opens door #3 [which in turn always reveals a goat]".

So statistically speaking one can arrive at the correct answer to Hypo A by considering and properly answering the following [after being told the same prize-door facts; that there are three doors and behind one is a desired car]:

> If you were a contestant on the aforementioned game show and in each round you always chose door **#1** and the Host always opened door **#2** to reveal a goat and the only possible change in fact pattern or possibility was that the car alternated (equally/randomly) between doors **#1** and **#3**, would you benefit statistically by deciding to switch to door **#3** in every round? [Of course not]

Or, if one wishes to get nitpicky about it, there are technically two other possible versions of the [substantially same] question that could alternatively be posed [not to be confused with being utilized in alternative statistical modeling, lest we end up back where we started], neither of which convey any meaningful or relevant information whatsoever in addition to or different from the preceding question, but for the purposes of thoroughness are worth spelling out:

> If you were a contestant on the aforementioned game show and in each round you always chose door **#2** and the Host always opened door **#3** to reveal a goat and the only possible change in fact pattern or possibility was that the car alternated (equally/randomly) between doors **#1** and **#2**, would you benefit statistically by deciding to switch to door **#1** in every round?

and

> If you were a contestant on the aforementioned game show and in each round you always chose door **#3** and the Host always opened door **#1** to reveal a goat and the only possible change in fact pattern or possibility was that the car alternated (equally/randomly) between doors **#3** and **#2**, would you benefit statistically by deciding to switch to door **#2** in every round?

In the event it is not clear, the only change in these three versions is "cosmetic"; that is, the change in the label (in terms of door number) given to each of the "choices". So we continue the analysis with the first of the three in mind.

Accordingly, we **always** know that the car **must** and will **always** be behind either door #1 or #3 [for they are the only non-Host revealed and thus putative non-goat doors]. And, as illustrated above, it does not matter whether we *substitute* the label or number associated with either the Contestant's or the Host's door "choice" [pairing] [these are key terms, because it's not really a choice at all; facts and events which have already transpired do not represent choices or contingencies, but rather conditions]. The car will still **always** be found behind one of only two doors (the non-Host door, in case it isn't clear). And since we are told and know that the car will **never** be behind the Host-door, the **only** persistent variable is which of the two remaining doors conceals it. What this means for Hypo A, therefore, is that there will **always** be a 50% chance of choosing - **initially and or subsequently** [i.e. pre or post switch] - correctly, and so **<u>the switch decision is statistically irrelevant</u>**.

What seems to confuse most people is the difference between a narrative that does *not* allow for meaningful statistical variation and one that does. Therefore, unlike the narrative contained in Hypo A, when, as in Hypo B, a scenario is presented in which greater relevant variation is introduced, we can see that the odds of being correct do indeed increase [to 66.6% or 2/3] when one always decides to switch. The set of possible variations of Hypo B can then be properly described as follows:

In round **1**, Contestant chooses door #**1**, Host opens door **#2** and car is behind #**3**;
In round **2**, Contestant chooses door #**1**, Host opens door **#2** and car is behind #**1**;
In round **3**, Contestant chooses door #**1**, Host opens door **#3** and car is behind #**2**;
In round **4**, Contestant chooses door #**2**, Host opens door **#1** and car is behind #**2**;
In round **5**, Contestant chooses door #**2**, Host opens door **#3** and car is behind **#**2;
In round **6**, Contestant chooses door #**2**, Host opens door **#1** and car is behind #**3**;
In round **7**, Contestant chooses door #**3**, Host opens door **#1** and car is behind #**2**;
In round **8**, Contestant chooses door #**3**, Host opens door **#2** and car is behind #**3**;
In round **9**, Contestant chooses door #**3**, Host opens door **#2** and car is behind #**1**.

These are all the possible scenarios that can play out when the facts of Hypo B are true (and there is no need to illustrate or interpret the statistics any further with regard to spelling out what happens with the decision to switch versus not switching, because I am not disputing the fact that the decision to switch in the foregoing scenario is statistically beneficial]. But when the narrative facts of Hypo A are true, **only** the following scenarios are possible:

In round 1, Contestant chooses door #**1**, Host opens door #**2** and prize is behind #**3**
[incorrect initial choice, thus switching leads to prize win];

and

In round 2, Contestant chooses door #**1**, Host opens door #**2** and prize is behind #**1**
[correct choice, thus switching leads to prize loss].

That's it. Period. Why? Because again, if we scrutinize the language in the narrative of Hypo A, it is clear that the *same* initial door "choice" must and will, *ipso facto*, *always* be made by Contestant. And the *same* door "choice" [opening] must and will, *ipso facto*, *always* be made by the Host. The narrative of Hypo A embodies the Groundhog Day version of the game show; each day - that is, each round, will necessarily produce, time and time again, the exact same "choices" of the Contestant and Host. Indeed, the narrative and factual content of Hypo A is fundamentally no different in terms of mathematical statistics than the following:

<u>Hypothetical A(1)</u>:

> You are a contestant on a game show. Behind one of three doors is a new car. Behind the other two doors are goats. For whatever reason - be it fate, compulsion, obsession or your affinity for Bill Murray - you **always and forever** choose door #1 initially. Thereafter, the Host **always and forever** - for the same quirky reasons - opens door #2 to **always and forever** reveal a goat. You are then asked….?

As can readily be seen now that the narrative language has been "converted" or translated to more straightforward content, it ought to be immediately apparent that the probability of choosing correctly remains the same whether or not a switch is made. Hence, it is fair and accurate to say that the so-called "Three-Door Problem" is really (and **always and forever**) a **two**-door problem in disguise.

In fact, the narrative of the Problem is almost always presented in a manner and with the language consistent with Hypo A hereinabove. Never have I ever heard of or seen it presented with reference to a Contestant who is "going to be" or "will in the future appear" on a game show; invariably the questioner asks you to "[I]magine you *are* a contestant on a game show…."

There is yet a third set of narrative possibilities ["Hypo C" group], wherein the narrative - particularly if conveyed verbally - is presented rather ambiguously, and, as stated above, it is in this context that most of the debate and confusion arises. And it is important to note that often the Problem presenter himself does not know or realize that he is presenting an inherently problematic narrative, incapable of resolution. We might call this Hypo C narrative conditional set the "Ship Barber" set, which takes its name from a riddle my father presented to me when I was a young boy, probably no more than five or six years old at the time.

It goes generally something like this: "There is a ship full of sailors, one of whom is the barber for the entire crew. And his rule is that he cuts [or shaves] the hair of everyone on the ship who does not cut [shave] their own hair. Who then, cuts the barber's hair? After mulling that over for a while you'll probably be left feeling like those who are presented with the narratives of Hypo C; there is no answer to the riddle, for, like Hypo C's, it contains illogical premises. Similarly, it may be impossible for the listener/Contestant to determine the "correct" answer based on the limited facts presented, in which case there is of course no "correct" answer. Hence the "Problem" is often one solely [and perhaps intentionally] attributable to linguistic, semantic and narrative shortcomings.

Going back to the Three Door Problem, after debating the matter with others I am often asked to walk through the statistical "factual" assumptions underpinning the problem, and inevitably the first thing - which the other person, failing to see the too-obvious-to-be-seen error of his or her own immediate assumptions, is certain I will agree with the first premise, which is usually phrased as something along the lines of, "Okay, you agree that there is a 1/3 (33%) chance at the outset with respect to door choice and prizes, right?". **Wrong**. In fact the mathematician(s) behind the "Ask Dr. Math" website that I reference below commits precisely the same error - and the narrative of the problem as presented therein virtually mirrors that of Hypo A, buttressing the correctness of my belief that the Problem is almost always presented with either identical or mathematically indistinguishable facts. *See* **www.mathforum.org/dr.math/faq/faq.monty.hall.html.**